Equilibrium refinements for repeated games

- SPNE is itself a refinement of NE. However, in repeated games (especially in(de)finitely repeated games) there are „too many” SPNE (see Folk Theorems).
- One way of dealing with it is to assume that players will focus on the best (Pareto Perfect) and symmetric (equitable) **outcomes** as in the Rottemberg-Saloner model
- Some, however, call for further refinements of the SPNE. Today we will discuss 3 such refinements:
  - Subgame Pareto Perfection
  - Weak Renegotiation Proofness
  - Markov Perfection

(Subgame) Pareto Perfection

- (Subgame) Pareto Perfection (Bernheim, Peleg and Whinston): An SPNE is SPP if strategies used by players are not Pareto dominated in any subgame (continuation game), i.e. if there is no other pair of strategies that form a NE and Pareto dominate the given strategies
  - Notice that the ‘nice’ SPNE in the 2-times repeated 3x3 game (with R played in stage 1) is not SPP, because in the subgame, where players are supposed to ‘punish’, (L,L) is dominated by another Nash equilibrium (M,M)
  - Also, all equilibria that involve trigger strategies in infinitely repeated games are eliminated

(Subgame) Pareto Perfection

- The informal argument is „renegotiation proofness”: if a deviation indeed occurred, the players would prefer to „re-negotiate” the equilibrium and play (M,M) instead of (L,L) (or (p^m, p^m) instead of (c, c))
- Notice the paradox: trying to select the Pareto efficient strategies results in eliminating the most efficient equilibrium (because a punishment tool is eliminated)

Weak Renegotiation Proofness

- Weak Renegotiation Proofness (Farrell&Maskin):an equlibirum is WRP if in every subgame the continuation payoffs are not Pareto dominated by another continuation payoff attainable **in this same equilibrium.**
- Works only for in(de)finitely repeated games. The argument: players can re-negotiate and switch to a different „subgame” within the bounds of the same equilibrium
- Eliminates ‘nice’ equilibria based on trigger strategies

Nice WRP equilibria

- Consider the infinitely repeated version of the Advertising Game
- Suppose that players play the following strategies:
  - Start with cooperation *"N";*
  - If player *i* deviates, switch to punishment phase;
  - In the punishment phase, player *i* (defector) plays *"N",* but the other player plays *"A";*
  - Punishment stops (both return to cooperation) once player *i* plays *"N".*

“carrot-and-stick”

- The above strategies, called ‘tit-for-tat’ have the „carrot-and-stick” property:
  - Are forgiving: nice behavior is rewarded
  - Are provokable: bad behavior is punished immediately
- Strategies of this kind are very robust. Not only do they survive various equilibrium refinements, but also do well in real-life games

Axelrod’s tournaments

- In 1980 Axelrod designed an experiment. Game theorists were asked to submit a strategy for a 200-times repeated Prisoner’s Dilemma (Advertising Game). The winning strategy (out of 14) in the first edition of the experiment was the strategy “Tit-for-Tat”.

Markov perfection

- Markov perfection is a strong refinement on the space of strategies that players can use.
- In markov-perfect equilibria (MPE), the decision (action) made in any period can depend only on a payoff-relevant ‘state variable’ *s*, which summarizes the history of the game, and not on the full history.
- Examples:
  - Resource extraction game: *s* is the amount of resource remaining
  - Bequest game: *s* is the amount of capital accummulated until now

MPE

- The idea behind MPE is that ‘bygones are bygones’, we should disregard all payoff-irrelevant information when making a decision
- MPE are not very useful (too strong) in simpler games. Notice that in the infinitely repeated Bertrand game (Advertising game) the only MPE is for both players to choose $p=c$ (advertise) in each period.